

Assume the theorem is false. We know the following:

$$(\forall v_0) \text{ sv}_1(v_0) = \text{sv}_2(v_0) \implies \text{SINGLEVALUED}(v_0) \quad (1)$$

$$(\forall v_0, v_1) \langle v_0, v_1 \rangle \in \text{POWER} \implies \mathcal{P}(v_0) = v_1 \quad (2)$$

We then substitute v_{64} for v_0 to obtain

$$(\forall v_1, v_{64}) \langle v_{64}, v_1 \rangle \in \text{POWER} \implies \mathcal{P}(v_{64}) = v_1 \quad (3)$$

In $(\forall v_0) \text{ U}(\mathcal{P}(v_0)) = v_0$ we substitute v_{64} for v_0 to obtain

$$(\forall v_{64}) \text{ U}(\mathcal{P}(v_{64})) = v_{64}$$

which together with (3) yields

$$(\forall v_1, v_{64}) \langle v_{64}, v_1 \rangle \in \text{POWER} \implies \text{U}(v_1) = v_{64}$$

We then substitute v_0 for v_{64} to obtain

$$(\forall v_0, v_1) \langle v_0, v_1 \rangle \in \text{POWER} \implies \text{U}(v_1) = v_0 \quad (4)$$

We know the following:

$$(\forall v_0) v_0 \subseteq V \times V \wedge \text{SINGLEVALUED}(v_0) \implies \text{FUNCTION}(v_0)$$

We then substitute v_{64}^{-1} for v_0 to obtain

$$(\forall v_{64}) v_{64}^{-1} \subseteq V \times V \wedge \text{SINGLEVALUED}(v_{64}^{-1}) \implies \text{FUNCTION}(v_{64}^{-1}) \quad (5)$$

We know the following:

$$(\forall v_0) v_0^{-1} \subseteq V \times V$$

We then substitute v_{64} for v_0 to obtain

$$(\forall v_{64}) v_{64}^{-1} \subseteq V \times V$$

which together with (5) yields

$$(\forall v_{64}) \text{ SINGLEVALUED}(v_{64}^{-1}) \implies \text{FUNCTION}(v_{64}^{-1})$$

We then substitute v_0 for v_{64} to obtain

$$(\forall v_0) \text{ SINGLEVALUED}(v_0^{-1}) \implies \text{FUNCTION}(v_0^{-1}) \quad (6)$$

We know the following:

$$(\forall v_0) \text{ SINGLEVALUED}(v_0) \vee \langle \text{sv}_3(v_0), \text{sv}_1(v_0) \rangle \in v_0$$

We then substitute v_0^{-1} for v_0 to obtain

$$(\forall v_0) \text{ SINGLEVALUED } (v_0^{-1}) \vee \langle \text{sv}_3 (v_0^{-1}), \text{sv}_1 (v_0^{-1}) \rangle \in v_0^{-1}$$

which together with (6) yields

$$(\forall v_0) \text{ FUNCTION } (v_0^{-1}) \vee \langle \text{sv}_3 (v_0^{-1}), \text{sv}_1 (v_0^{-1}) \rangle \in v_0^{-1} \quad (7)$$

We know the following:

$$(\forall v_0) \text{ SINGLEVALUED } (v_0) \vee \langle \text{sv}_3 (v_0), \text{sv}_2 (v_0) \rangle \in v_0$$

We then substitute v_0^{-1} for v_0 to obtain

$$(\forall v_0) \text{ SINGLEVALUED } (v_0^{-1}) \vee \langle \text{sv}_3 (v_0^{-1}), \text{sv}_2 (v_0^{-1}) \rangle \in v_0^{-1}$$

which together with (6) yields

$$(\forall v_0) \text{ FUNCTION } (v_0^{-1}) \vee \langle \text{sv}_3 (v_0^{-1}), \text{sv}_2 (v_0^{-1}) \rangle \in v_0^{-1} \quad (8)$$

In (4) we substitute $\mathcal{P}(v_{64})$ for v_1 to obtain

$$(\forall v_0, v_{64}) \langle v_0, \mathcal{P}(v_{64}) \rangle \in \text{POWER} \implies \text{U}(\mathcal{P}(v_{64})) = v_0$$

which together with $(\forall v_{64}) \text{U}(\mathcal{P}(v_{64})) = v_{64}$ yields

$$(\forall v_0, v_{64}) \langle v_0, \mathcal{P}(v_{64}) \rangle \in \text{POWER} \implies v_0 = v_{64}$$

We then substitute v_1 for v_{64} to obtain

$$(\forall v_0, v_1) \langle v_0, \mathcal{P}(v_1) \rangle \in \text{POWER} \implies v_0 = v_1 \quad (9)$$

We know the following:

$$(\forall v_0) \text{ FUNCTION } (v_0) \wedge \text{FUNCTION } (v_0^{-1}) \implies \text{ONEONE} (v_0)$$

We then substitute POWER for v_0 to obtain

$$\text{FUNCTION} (\text{POWER}) \wedge \text{FUNCTION} (\text{POWER}^{-1}) \implies \text{ONEONE} (\text{POWER})$$

which together with $\text{FUNCTION} (\text{POWER})$ yields

$$\text{FUNCTION} (\text{POWER}^{-1}) \implies \text{ONEONE} (\text{POWER}) \quad (10)$$

In (7) we substitute POWER for v_0 to obtain

$$\text{FUNCTION} (\text{POWER}^{-1}) \vee \langle \text{sv}_3 (\text{POWER}^{-1}), \text{sv}_1 (\text{POWER}^{-1}) \rangle \in \text{POWER}^{-1}$$

which together with (10) yields

$$\text{ONEONE}(POWER) \vee \langle \text{sv}_3(POWER^{-1}), \text{sv}_1(POWER^{-1}) \rangle \in POWER^{-1}$$

which together with $\neg \text{ONEONE}(POWER)$ yields

$$\langle \text{sv}_3(POWER^{-1}), \text{sv}_1(POWER^{-1}) \rangle \in POWER^{-1} \quad (11)$$

We know the following:

$$(\forall v_0, v_1, v_2) \langle v_0, v_1 \rangle \in v_2^{-1} \implies \langle v_1, v_0 \rangle \in v_2 \quad (12)$$

We then substitute $\text{sv}_3(POWER^{-1})$ for v_0 and $\text{sv}_1(POWER^{-1})$ for v_1 and $POWER$ for v_2 to obtain

$$\langle \text{sv}_3(POWER^{-1}), \text{sv}_1(POWER^{-1}) \rangle \in POWER^{-1} \implies \langle \text{sv}_1(POWER^{-1}), \text{sv}_3(POWER^{-1}) \rangle \in POWER$$

which together with (11) yields

$$\langle \text{sv}_1(POWER^{-1}), \text{sv}_3(POWER^{-1}) \rangle \in POWER \quad (13)$$

In (8) we substitute $POWER$ for v_0 to obtain

$$\text{FUNCTION}(POWER^{-1}) \vee \langle \text{sv}_3(POWER^{-1}), \text{sv}_2(POWER^{-1}) \rangle \in POWER^{-1}$$

which together with (10) yields

$$\text{ONEONE}(POWER) \vee \langle \text{sv}_3(POWER^{-1}), \text{sv}_2(POWER^{-1}) \rangle \in POWER^{-1}$$

which together with $\neg \text{ONEONE}(POWER)$ yields

$$\langle \text{sv}_3(POWER^{-1}), \text{sv}_2(POWER^{-1}) \rangle \in POWER^{-1} \quad (14)$$

In (2) we substitute $\text{sv}_1(POWER^{-1})$ for v_0 and $\text{sv}_3(POWER^{-1})$ for v_1 to obtain

$$\langle \text{sv}_1(POWER^{-1}), \text{sv}_3(POWER^{-1}) \rangle \in POWER \implies \mathcal{P}(\text{sv}_1(POWER^{-1})) = \text{sv}_3(POWER^{-1})$$

which together with (13) yields

$$\mathcal{P}(\text{sv}_1(POWER^{-1})) = \text{sv}_3(POWER^{-1})$$

which together with (14) yields

$$\langle \mathcal{P}(\text{sv}_1(POWER^{-1})), \text{sv}_2(POWER^{-1}) \rangle \in POWER^{-1} \quad (15)$$

In (12) we substitute $\mathcal{P}(\text{sv}_1(POWER^{-1}))$ for v_0 and $\text{sv}_2(POWER^{-1})$ for v_1 and $POWER$ for v_2 to obtain

$$\langle \mathcal{P}(\text{sv}_1(POWER^{-1})), \text{sv}_2(POWER^{-1}) \rangle \in POWER^{-1} \implies \langle \text{sv}_2(POWER^{-1}), \mathcal{P}(\text{sv}_1(POWER^{-1})) \rangle \in POWER$$

which together with (15) yields

$$\langle \text{sv}_2(\text{POWER}^{-1}), \mathcal{P}(\text{sv}_1(\text{POWER}^{-1})) \rangle \in \text{POWER} \quad (16)$$

In (9) we substitute $\text{sv}_2(\text{POWER}^{-1})$ for v_0 and $\text{sv}_1(\text{POWER}^{-1})$ for v_1 to obtain

$$\langle \text{sv}_2(\text{POWER}^{-1}), \mathcal{P}(\text{sv}_1(\text{POWER}^{-1})) \rangle \in \text{POWER} \implies \text{sv}_2(\text{POWER}^{-1}) = \text{sv}_1(\text{POWER}^{-1})$$

which together with (16) yields

$$\text{sv}_2(\text{POWER}^{-1}) = \text{sv}_1(\text{POWER}^{-1}) \quad (17)$$

In (1) we substitute POWER^{-1} for v_0 to obtain

$$\text{sv}_2(\text{POWER}^{-1}) = \text{sv}_1(\text{POWER}^{-1}) \implies \text{SINGLEVALUED}(\text{POWER}^{-1})$$

which together with (17) yields

$$\text{SINGLEVALUED}(\text{POWER}^{-1})$$

In (6) we substitute POWER for v_0 to obtain

$$\text{SINGLEVALUED}(\text{POWER}^{-1}) \implies \text{FUNCTION}(\text{POWER}^{-1})$$

which together with $\text{SINGLEVALUED}(\text{POWER}^{-1})$ yields

$$\text{FUNCTION}(\text{POWER}^{-1})$$

which together with (10) yields

$$\text{ONEONE}(\text{POWER})$$

but this contradicts $\neg \text{ONEONE}(\text{POWER})$.